RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIFTH SEMESTER EXAMINATION, DECEMBER 2018
THIRD YEAR [BATCH 2016-19]
PHYSICS [Honours]
Paper : V [Gr-C & D]

Date : 18/12/2018
Time : 11 am – 1 pm
Full Marks : 50

Group – C

(***Answer <u>any three</u> questions***) [3×10]

1. a) State the principle of superposition in quantum mechanics. What is the basic characteristic of Schrödinger's equation which leads to this principle? (2)

   b) If $\hat{A}, \hat{B}$ and $\hat{C}$ are three linear operators, prove that $\left[\hat{A},\left[\hat{B},\hat{C}\right]\right]+\left[\hat{B},\left[\hat{C},\hat{A}\right]\right]+\left[\hat{C},\left[\hat{A},\hat{B}\right]\right]=0$

   What can you say about the simultaneous measurement of the dynamical variables $\hat{A}$ and $\hat{B}$ if $\left[\hat{A},\hat{B}\right]=0$? (3)

   c) An atom in an excited state has a lifetime of $1.2\times10^{-8}s$ in a second excited state the lifetime is $2.3\times10^{-8}s$. What is the uncertainty in energy for the photon emitted when an electron makes a transition between there two levels? (2)

   d) The wave function of a particle is given as $\psi(x)=\frac{1}{\sqrt{a}}e^{-|x|/a}$. Find the value of b so that the probability of finding the particle in the range $-b<x<b$ is 0.5. (3)

2. a) Prove that if during Compton scattering the recoil electron energy is very low (non-relativistic) then the change in wavelength of the scattered photon is given by $\Delta\lambda=\frac{\lambda_c}{2}\left(\frac{\lambda'}{\lambda}+\frac{\lambda}{\lambda'}-2\cos\phi\right)$ (symbols have their usual meaning). (3)

   b) Show that if for a one-dimensional potential $V(-x)=V(x)$, the eigen functions of the Schrödinger equation are either symmetric or anti-symmetric function of *x*. (Assume non-degeneracy). (2)

   c) Show that if ψ be an eigenfunction of the operator $\hat{A}$ with eigenvalue λ , then it is also eigenfunction of $e^{\hat{A}}$ with eigenvalue e^{λ}. (2)

   d) Calculate the de Broglie wavelengths of electrons of total energies 2 MeV and 20 MeV respectively. (3)

3. a) The eigenfunctions for a potential of the form

$$V(x)=\alpha \qquad x<0;\ x>a$$
$$=0 \qquad 0<x<a$$

   are of the form $u_n(x)=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

   Suppose a particle in the preceding potential has an initial normalized wave function of the form $\psi(x,0)=A\left(\frac{\sin\pi x}{a}\right)^3$.

   (i) What is the form of $\psi(x,t)$? (2½)

   (ii) Calculate *A*. (1½)

   (iii) What is the probability that an energy measurement yields E_3, where $E_n=\frac{n^2\pi^2\hbar^2}{2ma^2}$. (3)

b) Show that $\frac{d}{dt}\langle x \rangle = \frac{\langle P_x \rangle}{m}$, where the symbols have their physical meanings. (3)

4. a) A particle of mass m moves in one dimension under the influence of a potential $V(x)$. Suppose it has an eigenstate $\psi(x) = \left(\frac{\alpha^2}{\pi}\right)^{1/4} exp\left(-\frac{\alpha^2 x^2}{2}\right)$ of the Hamiltonian with energy $E = \frac{\hbar^2 \alpha}{2m}$. Find the potential $V(x)$. (3)

b) Define bound states in quantum mechanics (consider one dimension only) along with the relation between the energy E and the potential required for such states to occur.
Which of the following problems admit of the unbound states? (a) infinite well, (b) finite well? (Give a brief explanation in each case). (3+2)

c) Estimate the zero point energy for a neutron in a nucleus, by treating it as if it were in an infinite square well of with equal to a nuclear diameter of 10^{-14} m. (2)

5. A hydrogen atom is, at time $t = 0$, in a state given by the wave function [$\psi_{nlm} \rightarrow$ Hydrogen wave function] $\psi(\vec{r},0)\frac{1}{\sqrt{10}}\left(2\psi_{100} + \psi_{210} + \sqrt{2}\psi_{211} + \sqrt{3}\psi_{21-1}\right)$

a) Find the expectation value of the Hamiltonian operator in this state. (2)

b) Is the state degenerate? Does the state have a definite parity? What is the expectation value of the angular momentum (orbital) in the state? What value of the component of spin along any arbitrary direction of the electron will you obtain as a result of measurement? Will you obtain bound states only for an electron in a Coulomb potential. (1+1+2+2+2)

<u>Group – D</u>

(***Answer <u>any two</u> questions***) [2×10]

6. a) Describe briefly Stern-Gerlach experiment and explain its significance. (2+2)

b) What do you mean by spontaneous emission and stimulate emission? (2)

c) Find the energies of the spectral lines emitted in the transition ${}^2P_{1/2} \rightarrow {}^2S_{1/2}$ when a hydrogen atom is placed in a weak magnetic field. (4)

7. a) Define LS Coupling. What are the basic differences between the hydrogen spectrum and the alkali spectrum. (2+2)

b) Which of the following molecules will show rotational-vibrational spectrum. Explain your answer. H_2, CO, O_2, HF. (2)

c) The force constant of the bond in CO molecule is 1870Nm^{-1}. Calculate the frequency of vibration of the molecule and the spacing between its vibrational energy levels in eV. (4)

8. a) What is Raman effect? Discuss the classical theory of Raman effect. (1+3)

b) In the vibrational Raman spectrum of $H\,F$ molecule, the Raman lines are observed at wavelength 2670 $\mathring{A}$ and 3430 $\mathring{A}$. Find the fundamental vibrational frequency of the molecule. (3)

c) If one hydrogen atom in a hydrogen molecule is replaced by a deuterium atom, will there be any change in (i) rotational constant, (ii) interatomic separation, justify your answer. (2+1)

9. a) Write down the rate equations of three level laser system, clearly. (3)

b) Deduce the expression for the Lande-g factor. Find out the change in energy when an atom is placed in a weak magnetic field. (2½+1½)

c) A 3mW laser beam of wavelength 600 nm and beam width ($2a$) of 1cm, incident on a lens of the focal length of 5 cm. Calculate the intensity and peak value of the electric field on the focused spot. ($\epsilon_0 = 8.854\times10^{-12} F/m$). (1+1)

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